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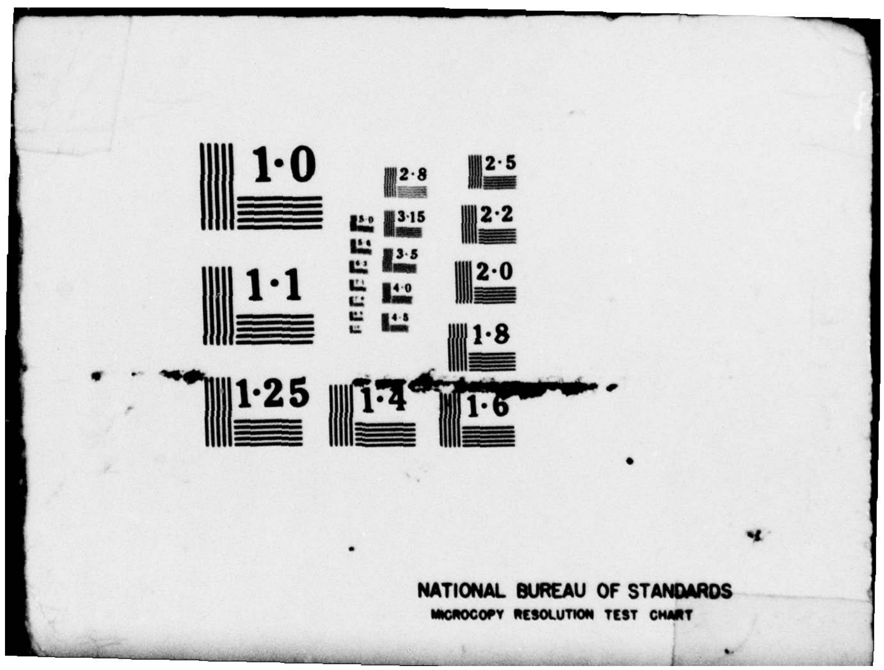
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THE LOGIT METHOD FOR COMBINING TESTS

1979

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E. OLUSEGUN GEORGE and GOVIND S. MUDHOLKAR*

ABSTRACT

A new combination statistic, the sum of the logits of P-values is introduced and its standardized form is found to be essentially equivalent to a standardized Student t in null distribution. Comparisons with Fisher's omnibus procedure by Bahadur ARE, membership in a complete class of tests and a Monte Carlo simulation of the power functions, show that both procedures are equivalent by the first criterion and neither procedure is generally dominant by ~~their~~ either of the two criteria.

LEVEL II

Key words: Combining tests, Logit, Fisher's Omnibus procedure, Bahadur Efficiency, admissibility.

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1. INTRODUCTION

Several authors have studied the problem of combining independent tests, especially noteworthy among whom are Fisher (1932), Karl Pearson (1933) E.S. Pearson (1938), Wallis (1942), A. Birnbaum (1954), Liptak (1958), van Zwet and Oosterhoff (1967), Oosterhoff (1969), and Littell and Folks (1971, 1973). The combination problem has been cast and analyzed in various canonical forms by these authors, but for the purposes of this essay its essentials may be described as follows: Let T_i, $i=1,2,\ldots,k$ be k independently distributed statistics, the large values of which are significant in testing respective null hypotheses H_{i0}: $\theta_i = \theta_{i0}$ against one-sided alternatives H_{i1}: $\theta_i > \theta_{i0}$ concerning real-valued parameters θ_i of the distributions of T_i, $i=1,2,\ldots,k$. The problem is to find a reasonable combination, i.e. a function of $T_1, T_2, \ldots, T_k$ which may be used for testing the overall hypothesis H_0: $\theta_i = \theta_{i0}$, $i=1,2,\ldots,k$ versus the alternative H_1: $\theta_i \geq \theta_{i0}$, $i=1,2,\ldots,k$ with at least one inequality strict. Many problems of combining tests of composite hypotheses, such as F-test of the general linear hypothesis, can be reduced to this form.

In describing the combination problem it is often convenient to use the P-values P_i of the tests as the pivotal entities instead of the statistics T_i; a combination statistic is then a function of $P_1, P_2, \ldots, P_k$. The P-values P_i is of course the probability, under the null hypothesis, of obtaining at least as extreme a value of T_i as observed. Thus denoting the null distribution of T_i by F_{i0} the P-value is given by

$$\begin{aligned} P_i &= 1-F_{i0}(T_i), \quad \text{if large values of } T_i \text{ are significant} \\ &= F_{i0}(T_i), \quad \text{if small values of } T_i \text{ are significant.} \end{aligned} \tag{1.1}$$

An advantage of using P_i is that it can be interpreted by itself, without a reference to the distribution of T_i. Moreover, if F_{i0} is continuous the null distribution of P_i is uniform on (0,1). This latter property of the P-values makes the null distributions of combination statistic manageable. In fact, most common combination statistics in the literature have simple null distributions. Notable examples of such statistics are (i) $T^{(T)}=\min P_i$, due to Tippett (1931); (ii) $T^{(W)}=P_{(r)}$, the r^{th} largest of the k P-values, due to Wilkinson (1951); (iii) $T^{(F)}= -2\Sigma\log P_i$, due to Fisher; (iv) $T^{(P)}=-2\Sigma\log(1-P_i)$, due to Pearson; and (v) $T^{(N)} = \Sigma\alpha_i\Phi^{-1}(1-P_i)$, due to Liptak, where $\Phi(\cdot)$ is the standard normal d.f. and

$\alpha_i \geq 0$ are arbitrary weights. Under the overall null hypothesis H_o $T^{(T)}$ and $T^{(W)}$ have beta distributions, $T^{(F)}$ and $T^{(P)}$ are χ^2_{2k}-variates and the Liptak's statistic $T^{(L)}$ is normally distributed. Among these procedures Fisher's procedure, which rejects H_o when $T^{(F)}$ is large, has been shown by Littell and Folks (1973) to be most efficient with respect to Bahadur's measure of A.R.E. A forteriori, Littell and Folks (1971, 1973) demonstrated that Fisher's method has maximum Bahadur A.R.E. among all reasonable combination methods based upon the P-values.

Furthermore Birnbaum (1954) proved that Wilkinson's (1951) and Pearson's (1933) method are inadmissible and recommended the use of Fisher's method for combining statistics whose distributions belong to the exponential family.

In this paper we introduce a new combination statistic, the logit statistic $T_L = -\sum_{i=1}^{k} \log \left[P_i/(1-P_i) \right]$ and suggest rejecting H_o when it is large, We show that this procedure has the same Bahadur efficiency as Fisher's method and consequently, because of a result of Littell and Folks (1973), both procedures are optimal.

The Logit statistic, its exact null distribution and a simple t-approximation to the null distribution are given in section 2. In section 3 the exact Bahadur slope is computed. In section 4 we answer a question raised by Oosterhoff (1969, p.42) on the admissibility of Fisher's method and discuss some complete class results for both methods. In section 5 we describe a Monte Carlo simulation of the power functions of the procedures when combining Student t tests.

2. THE LOGIT PROCEDURE

Following Berkson's (1944) term for the log-odds ratio $\log[P/(1-P)]$ the combination statistic

$$T_L = -\sum_{i=1}^{k} \log[P_i/(1-P_i)] \tag{2.1}$$

is termed the Logit statistic. When the null distribution F_{i0} of T_i is continuous P_i is uniformly distributed on $(0,1)$. Consequently under the null hypothesis H_{i0} $-\log[P_i/(1-P_i)]$ is distributed according to the logistic distribution function

$$F(z) = [1 + \exp(-z)]^{-1}, \quad -\infty < z < \infty \tag{2.2}$$

and the Logit statistic is a sum of i.i.d. logistic variates under H_0. George and Mudholkar (1977) have shown that the exact null distribution, F_0 of T_L, is given by

$$1-F_0(z) = \sum_{p=0}^{k-1} \sum_{r=0}^{k-1-p} \sum_{m=1}^{p+r+1} (-1)^{r+1} A_{k,p} S_{p+r+1,m} \frac{z^r}{r} \binom{k-1}{p+r} (m-1)! \cdot [e^{-z}/(1-e^{-z})]^m \tag{2.3}$$

when k is even, and by

$$1-F_0(z) = \sum_{p=0}^{k-1} \sum_{r=0}^{k-1-p} \sum_{m=1}^{p+r+1} (-1)^{r+m+1} A_{k,p} S_{p+r+1,m} \frac{z^r}{r} \binom{k-1}{p+r} (m-1)! \cdot [e^{-z} (1+e^{-z})]^k \tag{2.4}$$

when k is odd; where $A_{k,p}$'s are computed from the following equations.

$$[\pi s/\sin \pi s]^k = A_{k,0} + A_{k,1}s + A_{k,2}s^2 + \ldots \tag{2.5}$$

and

$$(\pi s/\sin \pi s) = \sum_{m=0}^{\infty} (-1)^{m-1} [(2^{2m}-2)/(2m)!] B_{2m} (\pi s)^{2m} , \tag{2.6}$$

where B_{2m}'s are Bernoulli numbers, and $S_{n,m}$'s are Stirling numbers of the second kind. Thus when $k = 2$,

$$1-F_2(z) = z[e^{-z}/(1-e^{-z})]^2 + (z-1)[e^{-z}/(1-e^{-z})] \tag{2.7}$$

and when $k = 3$

$$1-F_3(z) = e^{-z}/(1+e^{-z}) - 2ze^{-z}/(1+e^{-z})^2$$

$$+ (z^2 + \pi^2)e^{-z}(1+e^{-z})/2(1+e^{-z})^3 \,. \tag{2.8}$$

George and Mudholkar (1977) have illustrated that the convolution of k logistic random variables, and hence the null distribution of the Logit statistic, is well approximated by the normal distribution with equal variance, that this approximation becomes very good when Edgeworth-corrected, and that a multiple of the student's t-distribution with the degrees of freedom obtained by considering the kurtosis provides a simple and an even better approximation. Specifically, for the null distribution of Logit statistic, it was proposed that

$$-\sum_{i=1}^{k} \log[P_i/(1-P_i)] \approx \frac{k(5k+2)}{3(5k+4)}\, t_{5k+4} \,. \tag{2.9}$$

In Table 1 the quality of this approximation is illustrated numerically for k=3.

3. BAHADUR A.R.E. OF THE LOGIT STATISTIC

The concept of Bahadur A.R.E. is well discussed in the literature (Bahadur 1971). Let T_n be a statistic for testing a null hypothesis $H_0: \theta \in \Theta_0$ against an alternative $H: \theta \in \Theta_1$. Assume, without loss of generality, that large values of T_n are significant, and let $T_n = t_n$ be observed. Then the rate of convergence to zero of the P-value $L_n(t_n) = \Pr\{T_n \geq t_n | H_0\}$ as $n \to \infty$ is a measure of the efficiency of the test T_n. Specifically suppose that for $i = 1,2$ and $\theta \in \Theta_1$, there exist positive numbers $c_i(\theta)$ such that for sequences of tests $\{T_n^{(i)}\}$ with corresponding P-values $\{L_n^{(i)}\}$

$$\lim_{n\to\infty} -\frac{1}{n} \operatorname{Log} L_n^{(i)} = c_i(\theta) \quad \text{a.s.} \quad [P_\theta] \tag{3.1}$$

Then the Bahadur A.R.E. of $\{T_n^{(1)}\}$ relative to $\{T_n^{(2)}\}$ is given by

$$\Psi_{12}(\theta) = c_1(\theta)/c_2(\theta) \quad . \tag{3.2}$$

Since a Bahadur slope is not always easy to compute, Bahadur (1971) gave the following result to facilitate its computation.

Theorem (3.1) (Bahadur). Suppose that

$$n^{-1/2} T_n \to b(\theta) \text{ a.s. } P_\theta, \theta \in \Theta_1, \tag{3.3}$$

where $-\infty < b(\theta) < \infty$ and

$$\lim_{n\to\infty} n^{-1} \log[1-F_n(\sqrt{n}\, t)] = -f(t) \tag{3.4}$$

for each t in some open interval I, where $F_n(\cdot)$ is the null distribution function of T_n, f is continuous on I and $\{b(\theta), \theta \in \Theta_1\} \subset I$. Then

$$\lim_{n\to\infty} n^{-1} \operatorname{Log} L_n \text{ exists}$$

and

$$c(\theta) = 2f(b(\theta)). \tag{3.5}$$

Theorem 3.2: For i = 1, ..., k, let $\{T_{n_i}^{(i)}\}$ be sequences of statistics for respectively testing H_{i0}: $\theta_i = \theta_{i0}$ against H_{i1}: $\theta_i > \theta_{i0}$, and let $\{L_{n_i}^{(i)}$ be the corresponding P-values. Assume that the $T_{n_i}^{(i)}$'s are independently distributed and that there exists a positive value function $c_i(\theta_i)$ defined for $\theta_i > \theta_{i0}$ such that

$$\lim_{n_i \to \infty} n_i^{-1} \log L_{n_i}^{(i)} = c_i(\theta_i)/2 \quad \text{a.s.} \tag{3.6}$$

Also assume that

$$\lim_{n_i \to \infty} n_i/n = \lambda_i \ , \tag{3.7}$$

where $nk = n_1 + \dots + n_k$. Then the exact slope of the Logit combination procedure for testing $H_0: \theta_i = \theta_{i0}$, $i = 1,\dots,k$ against $H_1: \theta_i \geq \theta_{i0}$, $i=1,\dots,k$ with at least one strict inequality, is given by

$$c_L(\theta_1,\dots,\theta_k) = \sum_{i=1}^{k} \lambda_i c_i(\theta_i) \ . \tag{3.8}$$

Proof: It is well known that if $\phi_n(T_n)$ is a strictly increasing and continuous function of T_n, then $\{\phi_n(T_n)\}$ and $\{T_n\}$ have the same exact slopes.

Hence let

$$T_n^{(L)*} = n^{-1/2}\, T_n^{(L)}$$

$$= n^{-1/2} \sum_{i=1}^{k} \log[(1 - L_{n_i}^{(i)})/L_{n_i}^{(i)}]$$

Then

$$n^{-1/2}T_n^{(L)*} = \sum_{i=1}^{k} n^{-1}\log(1-L_{n_i}^{(i)}) - \sum_{i=1}^{k} n^{-1}\log L_{n_i}^{(i)}$$

$$= \sum_{i=1}^{k} (n_i/n)n_i^{-1}\log(1-L_{n_i}^{(i)}) - \sum_{i=1}^{k}(n_i/n)n_i^{-1}\log L_{n_i}^{(i)}.$$

Clearly $\operatorname{Lim}_{n_i\to\infty} n_i^{-1}\log L_{n_i}^{(i)} = c_i(\theta_i)$ w.p.1 implies that

$$\operatorname{Lim}_{n_i\to\infty} n_i^{-1}\log(1 - L_{n_i}^{(i)}) = 0 \text{ w.p.1.}$$

Hence from (3.6) and (3.7), it follows that

$$\operatorname{Lim}_{n\to\infty} n^{-1/2}T_n^{(L)*} = \sum_{i=1}^{k}\lambda_i c_i(\theta_i) \quad \text{w.p.1}$$

Now let F_0^* denote the d.f of $T_n^{(L)*}$. Then

$$1 - F_0^*(t) = 1 - F_0(\sqrt{n}\,t) \quad -\infty < t < \infty, \tag{3.9}$$

where F_0 is the d.f. of $T_n^{(L)}$. By combining the expressions for d.f. of F_0 given by (2.3) and (2.4), it can easily be shown that

$$\begin{aligned} 1-F_0^*(\sqrt{n}\,z) &= [e^{-nz}/(1-e^{-nz})]h_1(nz) \quad \text{if } k \text{ is even,} \\ &= [e^{-nz}/(1+e^{-nz})]h_2(nz) \quad \text{if } k \text{ is odd,} \end{aligned} \tag{3.10}$$

where for $j = 1,2$

$$h_j(z) = 1 + p_{1,j}(z)[e^{-z}/(1+(-1)^j e^{-z})]^{r_{1,j}} + \dots$$

$$+ p_{s,j}(z)[e^{-z}/(1+(-1)^j e^{-z})]^{r_{s,j}},$$

$p_{r,j}(z)$, $1 \le r \le s$, are polynomials in z,

and $r_{m,j}, s$, $1 \leq r_{m,j} \leq k$, are positive integers.

From (3.10) we get

$$-\frac{1}{n}\log[1-F_0^{\star}(\sqrt{n}\, z)] = -z - \frac{1}{n}\{\log(1-e^{-nz}) + \log h_1(nz)\} \text{ if } k \text{ is even},$$

$$= -z - \frac{1}{n}\{\log(1+e^{-nz}) + \log h_2(nz)\} \text{ if } k \text{ is odd}.$$

Clearly $\lim_{n\to\infty} h_j(nz) = 1$. Hence

$$-\lim_{n\to\infty} \frac{1}{n}\log[1 - F_0^{\star}(\sqrt{n}\, z)] = z.$$

Using Theorem 3.1, we immediately get the exact slope of $T_n^{(L)\star}$ to be $\sum_{i=1}^{k} \lambda_i c_i(\theta_i)$. Littell and Folks (1971) have shown that under the conditions of Theorem 3.2, the exact slope of Fisher's procedure is $\sum_{i=1}^{k} \lambda_i c_i(\theta_i)$. Furthermore they showed (1973) that among "essentially" all combination procedures based on P-values, Fisher's procedure is optimal according to Bahadur A.R.E. We state a slightly more general version of this result, which holds for the Logit and Fisher's procedures.

Theorem 3.3: Let T_n be a combination statistic for testing H_0 against H_1 based on P-values $L_{n_1}^{(1)},\ldots,L_{n_k}^{(k)}$. Suppose that T_n satisfies the following conditions.

I. T_n is non-decreasing in each $L_{n_i}^{(i)}$, i.e.,

$$\ell_1' \leq \ell_1,\ldots,\ \ell_k' \leq \ell_k \text{ imply } T_n(\ell_1',\ldots,\ell_k') \leq T_n(\ell_1,\ldots,\ell_k).$$

In this case small values of T_n are significant.

II. T_n is non-increasing in each $L_{n_i}^{(i)}$, i.e.,

$$\ell_1 \leq \ell_1',\ldots,\ell_k \leq \ell_k' \text{ imply } T_n(\ell_1,\ldots,\ell_k) \geq T_n(\ell_1',\ldots,\ell_k').$$

In this case large values of T_n are significant.

Then the slopes $c(\theta_1,\ldots,\theta_k)$ of $\{T_n\}$ and $c_L(\theta_1,\ldots,\theta_k)$ of the Logit and Fisher's procedures satisfy the inequality $c(\theta_1,\ldots,\theta_k) \leq c_L(\theta_1,\ldots,\theta_k)$.

<u>REMARK</u>: Wieand (1976) has discussed conditions under which Bahadur's and Pitman's methods of comparing tests coincide. Under the conditions he stated both the Logit and Fisher's methods of combining tests are optimal with respect to Pitman's efficiency as well.

<u>TABLE 1</u>

THE EXACT AND APPROXIMATE* D.F.
OF STANDARDIZED LOGIT STATISTIC, k=3

x	Exact	Approximate	Error
.05	.5209	.5208	0.0001
.25	.6033	.6028	0.0005
.45	.6809	.6802	0.0007
.65	.7506	.7499	0.0007
1.00	.8486	.8482	0.0004
1.20	.8903	.8901	0.0002
1.45	.9291	.9291	0.0000
1.75	.9598	.9600	-0.0002
2.50	.9918	.9920	-0.0002
3.00	.9975	.9975	0.0000

*The approximation is based upon Equation 2.9.

4. ADMISSIBILITY AND COMPLETE CLASS RESULTS

Several researchers (A. Birnbaum 1954, 1955; van Zwet and Oosterhoff 1967; Oosterhoff 1969 and Brown, Cohen and Strawderman 1976) have discussed the question of admissibility and membership in complete classes of combination procedures. Birnbaum (1955) proved that if the distributions of the component test statistics T_i are one-parameter exponential families, a procedure for combining these statistics can be admissible only if its acceptance region is monotone and convex in the $(t_1, \ldots, t_k)$ space. He further showed that this condition is both necessary and sufficient for admissibility if for each i, T_i is normally distributed with a known variance. Oosterhoff considered the more general problem in which the distribution of the T_i,s have strict monotone likelihood ratio (MLR) properties, then monotone procedures are essentially complete. Brown et al showed that under the conditions stated by Oosterhoff monotone procedures are complete. We now compare the Logit and Fisher's procedures in the light of the above results.

First we answer a question raised by Oosterhoff (1969, page 42) and show that Fisher's procedure is admissible when the distributions of the component statistics are one-parameter exponential families.

Theorem 4.1: If for each i, the statistic T_i for testing H_{i0}: $\theta_i = \theta_{i0}$ against H_{1i}: $\theta_i > \theta_{i0}$ have densities that can be expressed in the canonical form

$$f_i(t_i) = h(t_i)\, c(\theta_i) e^{\theta_i t_i} . \tag{4.1}$$

$i = 1,\ldots,k$ and if the T_i's are independent, then Fisher's combination procedure is admissible for testing H_0: $\theta_i = \theta_{i0}, i=1,\ldots,k$ against H_1: $\theta_i \geq \theta_{i0}$, $i=1,\ldots,k$ with at least one inequality strict.

The following result given by Mudholkar (1969) is used in the proof.

Lemma 4.1: Let $g_i(x_i)$ be positive functions of x_i, $i=1,\ldots,k$. If $g_i(x_i)$ is a logconcave function of x_i for each i, then the set $A = \{(x_1,\ldots,x_k): \prod_{i=1}^{k} g_i(x_i) > c\}$ is convex for any positive real constant c.

Proof of theorem 4.1: It is easily shown that if the distribution of T_i is given by equation 4.1, then P-value corresponding to observed value $T_i = t_i$ is given by

$$P_i(t_i) = 1 - F_{i0}(t_i).$$

Consequently, the acceptance region

$$A_F = \{(t_1,\ldots,t_k)\colon -2\sum_{i=1}^{k}\log P_i(t_i) < c_F\},$$

where c_F is a constant, of the Fisher's procedure, may be expressed as

$$A_F = \{(t_1\ldots,t_k)\colon \prod_{i=1}^{k}(1-F_{i0}(t_i)) > e^{-c_F/2}\}$$

Now for any vector $(t_1'\ldots t_k')$ such that $t_i' < t_i$, $i=1,\ldots,k$, it is clear that $(t_1,\ldots,t_k) \in A_F$ implies that $(t_1'\ldots,t_k') \in A_F$. Hence A_F is a monotone acceptance region. Now since T_i has an exponential family distribution, $1-F_{i0}(t_i)$ is logconcave. Therefore by Lemma 4.1, A_F is convex. The result of Birnbaum (1955) then implies that Fisher's procedure is admissible in the above context.

It is clear that in the exponential family case the acceptance region

$$A_L = \{(t_i\ldots,t_k)\colon \sum_{i=1}^{k}\log[\{1-F_{i0}(t_i)\}/F_{i0}(t_i)] > c_L\}$$

where c_L is a constant is not convex, however, convexity of an acceptance region is not known to be sufficient for admissibility.

Consequently the Logit procedure cannot be ruled out as inadmissible in the context of exponential family of distributions. Furthermore, in the larger context of combining tests whose component statistics have M.L.R. or strict M.L.R. properties, results due to Brown et al (1976) can be used to show that both procedures belong to essentially complete or complete classes of tests. The combination of independent Student t test and independent F tests are particular cases of the M.L.R. families that are of practical importance.

We report here an empirical evidence that shows that in combining independent Student t tests, neither of the Logit or Fisher's procedures dominates the other uniformly.

5. A STUDY OF THE POWER FUNCTIONS BY SIMULATION

In this section the power functions of the Logit and Fisher's combination statistics are studied in the context combining Student t tests. Let t_{n_i-1}, $i=1,2$ be two independent Student t statistics with degrees of freedom n_1-1 and n_2-1 respectively. Let t_{n_i-1} be used to test H_{i0}: $\mu_i=0$ versus H_{1i}: $\mu_i>0$, where μ_i is the unknown mean of a normal population with unknown variance. Consider the problem of testing the combined null hypothesis H_0: $\mu_1=\mu_2=0$ against the alternative H_1: Either $\mu_1>0$ or $\mu_2>0$ by using the Logit statistic $T^{(L)} = \log\left[(1-P_1)/P_1\right] + \log\left[(1-P_2)/P_2\right]$ or the Fisher's statistic $T^{(F)} = -2\log P_1 - 2\log P_2$, where P_1 and P_2 are the P-values corresponding to t_{n_1-1} and t_{n_2-1} respectively.

Although both $T^{(L)}$ and $T^{(F)}$ have simple null distributions under H_0, their distributions under H_1 are complex, consequently it is futile to attempt to obtain their exact power functions. A reasonable estimates of the powers may be obtained from a Monte-Carlo experiment.

5.1 THE MONTE CARLO EXPERIMENT

For $i=1,2$ n_i independent standard normal variates are generated on the IBM 360/365 computer at the University of Rochester by using the McGill University random number package developed by a technique due to Marsaglia (1961) for generating standard normal deviates. $N(\mu_i, 1)$ deviates are obtained by adding μ_i to each generated variable. From these independent Student t statistics and their corresponding P-values are obtained.

The power functions of the Logit and Fisher's procedures are approximated from these statistics at levels of signifance $\alpha=.01$ and $.05$ and for sample sizes $n_1=n_2=5$ and $n_1=n_2=10$. The power of each procedure is estimated at $\mu_1=0.0\ (0.1)\ 1.6$ and $\mu_2 = 0.0\ (0.1)\ 1.6$ by the proportion of times the procedure rejects H_0 in 3000 trials at each (μ_1, μ_2).

5.2 RESULTS

Table 2 represents some estimates of the power functions of the Logit and Fisher's procedures. From the table it can be observed that the power of each procedure is ncreasing in μ_2 when μ_1 is held fixed and vice-versa; and that the Logit procedure is slightly superior to Fisher's procedure around the equiangular line $\mu_1=\mu_2$ while Fisher's procedure is superior elsewhere.

POWER FUNCTION OF THE LOGIT AND FISHER'S METHODS FROM A MONTE CARLO EXPERIMENT

μ_2	μ_1 = 0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
	$\alpha = .05$, $n_1 = n_2 = 5$								
1.6	.487	.493	.584	.663	.780	.855	.914	.958	.976
	.558	.558	.641	.678	.772	.839	.897	.943	.963
1.4	.423	.442	.516	.610	.711	.809	.874	.918	.948
	.472	.484	.545	.610	.700	.789	.853	.900	.935
1.2	.321	.352	.432	.538	.635	.744	.821	.870	.907
	.354	.379	.440	.524	.615	.717	.788	.852	.893
1.0	.254	.271	.353	.424	.539	.646	.724	.787	.850
	.274	.281	.357	.411	.511	.617	.698	.762	.833
0.8	.195	.217	.267	.357	.452	.555	.635	.712	.774
	.197	.215	.254	.340	.432	.535	.610	.694	.773
0.6	.120	.149	.189	.263	.358	.433	.534	.598	.688
	.125	.147	.182	.259	.337	.420	.525	.609	.699
0.4	.088	.103	.142	.195	.259	.331	.428	.515	.589
	.083	.106	.136	.189	.254	.333	.439	.533	.633
0.2	.053	.068	.098	.139	.204	.277	.342	.445	.508
	.054	.069	.097	.140	.209	.287	.372	.484	.581
0.0	.052	.056	.088	.126	.190	.263	.332	.395	.486
	.054	.054	.090	.129	.192	.275	.365	.455	.550

μ_2	μ_1 0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
	α = .01, $n_1 = n_2 = 5$								
1.6	.191	.205	.281	.339	.469	.578	.645	.757	.801
	.216	.217	.286	.336	.450	.441	.607	.717	.771
1.4	.143	.158	.226	.296	.391	.496	.572	.670	.727
	.151	.167	.227	.287	.370	.463	.535	.631	.683
1.2	.099	.122	.170	.247	.315	.404	.515	.578	.653
	.106	.128	.167	.228	.283	.381	.481	.541	.621
1.0	.070	.080	.121	.172	.231	.320	.414	.463	.555
	.070	.080	.115	.159	.210	.297	.397	.429	.527
0.8	.047	.058	.084	.135	.180	.256	.305	.391	.460
	.048	.059	.081	.124	.160	.230	.283	.373	.438
0.6	.032	.045	.054	.081	.132	.171	.228	.286	.353
	.034	.048	.052	.076	.120	.161	.210	.272	.348
0.4	.019	.024	.035	.050	.089	.127	.160	.203	.274
	.018	.023	.035	.048	.081	.120	.156	.206	.281
0.2	.009	.015	.027	.038	.058	.086	.124	.157	.203
	.009	.015	.024	.036	.055	.090	.122	.167	.220
0.0	.009	.011	.019	.032	.050	.073	.109	.143	.172
	.009	.012	.021	.032	.048	.081	.115	.152	.200

	μ_1								
μ_2	0.0	0.2	0.3	0.4	0.5	0.7	0.9	1.0	
	α = .05, n_1 = 10, n_2 = 10								
1.0	.575 .651	.595 .761	.612 .676	.675 .720	.737 .755	.790 .811	.904 .898	.952 .948	.963 .958
0.9	.487 .550	.518 .579	.546 .599	.592 .622	.683 .705	.732 .735	.853 .836	.927 .917	.954 .950
0.7	.339 .374	.353 .383	.387 .410	.451 .466	.539 .535	.598 .598	.754 .737	.850 .845	.895 .894
0.5	.199 .211	.201 .220	.240 .249	.292 .294	.362 .358	.440 .429	.808 .601	.721 .731	.789 .807
0.4	.151 .154	.165 .169	.183 .180	.237 .233	.287 .282	.358 .347	.502 .509	.663 .676	.726 .751
0.3	.100 .106	.121 .122	.143 .148	.175 .170	.228 .226	.297 .300	.458 .468	.597 .630	.666 .709
0.2	.080 .077	.075 .075	.102 .102	.143 .142	.181 .183	.240 .244	.388 .413	.543 .597	.626 .688
0.1	.056 .058	.062 .062	.078 .078	.108 .105	.146 .151	.211 .224	.360 .393	.534 .596	.573 .642
0.0	.052 .049	.052 .054	.071 .066	.101 .105	.148 .156	.203 .219	.338 .378	.495 .558	.557 .631

*Lower elements correspond to power of Fisher's method
Upper elements correspond to power of Logit method.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new combination statistic, the sum of the logits of P-values is introduced and its standardized form is found to be essentially equivalent to a standardized Student t in null distribution. Comparisons with Fisher's omnibus procedure by Bahadur ARE, membership in a complete class of tests and a Monte Carlo simulation of the power functions, show that both procedures are equivalent by the first criterion and neither procedure is generally dominant by either of the two criteria.

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